The Expected Fringe Shift in Ether-Drift Interferometry: A Worked Example

Anonymous submission for peer review

Abstract. This is an example paper prepared with the Natural Philosophy Journal LaTeX template. It derives the fringe shift that a stationary-ether theory predicts for a Michelson interferometer, applies the result to the 1887 Michelson–Morley apparatus, and compares the prediction with the upper bounds reported in later repetitions of the experiment. The central claim, stated so that it can be checked, is modest: under the stationary-ether hypothesis without length contraction, the 1887 apparatus should have shown a shift of about 0.4 of a fringe, and the reported bound is at least twenty times smaller. The paper is intended to show the layout of equations, tables, figures and references, not to advance a new result.

Keywords: Michelson–Morley experiment; interferometry; ether drift; length contraction; history of physics

1 Introduction

State the central claim early and plainly. Here it is: *if light travels at speed c relative to a stationary ether, and the interferometer arms keep their length when moving through it, then the 1887 Michelson–Morley apparatus should have shown a fringe shift of about* 0.4 *on rotation.* The experiment was designed to detect exactly this effect [5, 6].

The standard account, from which much of the work in this journal departs, interprets the null result through special relativity [1]. Earlier, FitzGerald and Lorentz had proposed that bodies moving through the ether contract along the direction of motion [2, 4]. Later observers, notably Miller, reported small non-null effects [7], which a subsequent analysis attributed to temperature gradients in the apparatus [8]. Section 2 derives the expected shift, Section 3 compares it with reported bounds, and Section 4 discusses what the comparison does and does not establish.

2 Theory

2.1 Light travel times

Figure 1 shows the arrangement. Light from the source is divided at the beam splitter into two beams, which travel along arms of equal length L and recombine to form fringes.

Definition 1. Let v be the speed of the apparatus relative to the ether, $\beta = v/c$, and $\gamma = (1-\beta^2)^{-1/2}$.

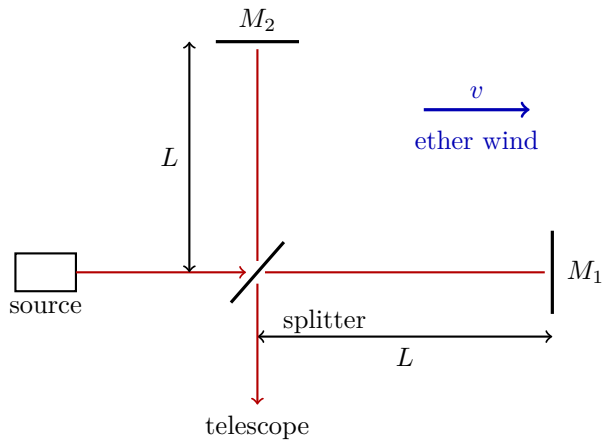

Figure 1: Schematic Michelson interferometer. The apparatus moves at speed v relative to the hypothetical ether, parallel to the arm ending at mirror M_1.

For the arm parallel to the motion, the light travels against the ether wind on the way out and with it on the way back:

$$t_{\parallel} = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c}\,\gamma^2. \tag{1}$$

For the perpendicular arm, the light must be aimed slightly upstream, so that its speed across the arm is $\sqrt{c^2 - v^2}$:

$$t_{\perp} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\,\gamma. \tag{2}$$

2.2 The expected shift

The difference between Eqs. (1) and (2), expanded to lowest order in β, is

$$\begin{aligned}\Delta t = t_{\parallel} - t_{\perp} &= \frac{2L}{c}\left(\gamma^2 - \gamma\right) \\ &\approx \frac{2L}{c}\left[\left(1+\beta^2\right) - \left(1+\tfrac{1}{2}\beta^2\right)\right] = \frac{L}{c}\,\beta^2.\end{aligned} \tag{3}$$

Rotating the apparatus through 90° exchanges the roles of the arms, so the observed change is $2\Delta t$. Expressed in fringes of wavelength λ,

$$N = \frac{2c\,\Delta t}{\lambda} = \frac{2L}{\lambda}\,\beta^2. \tag{4}$$

For an intermediate orientation θ of the parallel arm to the ether wind, the shift relative to its mean value varies as $\frac{1}{2}N\cos 2\theta$, completing two cycles per rotation (Fig. 2).

2.3 Numbers for the 1887 apparatus

Michelson and Morley lengthened the effective arm by multiple reflections to $L \approx 11$ m [6]. Taking the orbital speed of the Earth, $v \approx 30\ \mathrm{km\,s^{-1}}$, gives $\beta \approx 1.0 \times 10^{-4}$. With $\lambda \approx 550$ nm, Eq. (4) gives

$$N \approx \frac{2 \times 11\ \mathrm{m}}{5.5 \times 10^{-7}\ \mathrm{m}} \times \left(1.0 \times 10^{-4}\right)^2 \approx 0.40. \tag{5}$$

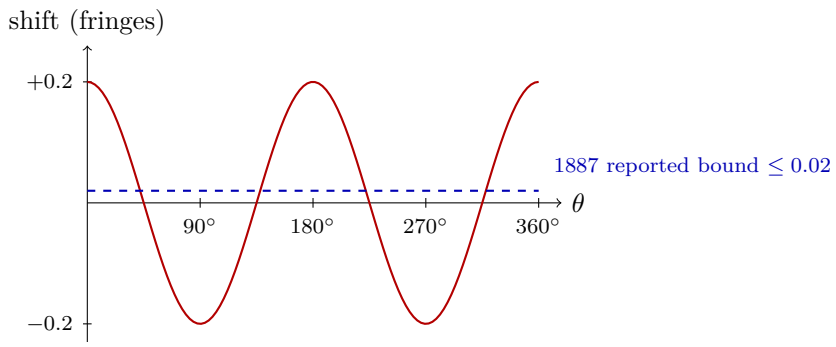

Figure 2: Predicted variation of the fringe position with orientation θ for the 1887 apparatus (solid), with $N = 0.4$ peak to peak, compared with the size of the reported upper bound (dashed).

3 Comparison with reported results

Table 1 sets the expected shift beside the upper bounds reported for several interferometers. Every reported value is accompanied by its bound, as the Author Guidelines require: a number without an uncertainty cannot be refereed.

Table 1: Expected and reported fringe shifts. Values are rounded and are given to illustrate table layout; check them against the original reports before relying on them.

Experiment	Arm length L (m)	Expected N	Reported shift
Michelson 1881 [5]	1.2	0.04	≤ 0.02
Michelson & Morley 1887 [6]	11.0	0.40	≤ 0.02
Miller 1921–1926 [7]	32.0	1.12	≤ 0.08
Joos 1930 [3]	21.0	0.75	≤ 0.002

In every case the reported shift is much smaller than the expected one. The ratio is about 20 for the 1887 experiment and larger for the later, more sensitive instruments.

4 Discussion

The comparison rules out the stationary-ether hypothesis *in the form assumed in Section 2*, that is, with rigid arms. It does not by itself decide between the explanations that remove the predicted shift. If the parallel arm contracts by the factor $1/\gamma$ [2, 4], Eq. (1) becomes $t_{\parallel} = (2L/c)\,\gamma = t_{\perp}$ and the shift vanishes exactly. Special relativity predicts the same null result [1], as do models in which the ether is entrained near the Earth's surface.

A reader who wishes to dispute the conclusion should address the strongest contrary evidence. Here that is Miller's report of a persistent periodic signal [7], and the later analysis attributing it to temperature effects [8]. A paper that relied on Miller's signal would need to answer that analysis directly.

5 Conclusions

Under a stationary ether with rigid arms, the 1887 apparatus should have shown a shift of about 0.4 fringe, and the reported shift was at least twenty times smaller. The calculation constrains

the rigid-arm ether model only; distinguishing between the theories that predict a null result requires other experiments.

A Exact form of the time difference

Without the small-β expansion used in Eq. (3),

$$\Delta t = \frac{2L}{c}\,\gamma\,(\gamma - 1)\,, \tag{6}$$

which reduces to $(L/c)\beta^2$ to lowest order. For $\beta = 10^{-4}$ the next term is smaller by a factor of order $\beta^2 = 10^{-8}$, so the approximation is more than adequate.

References

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